22

$y = x^3 - 3x^2 + 3x - 2$

$y' = 3x^2 - 6x + 3$

$y' = 3(x^2 - 2x + 1)$

$y' = 3(x-1)^2$

critical Point

$f'(x) = 0$

$f'(x) = \text{undef.}$

$0 = 3(x-1)^2$

$x = 1$ makes $f'(x) = 0$

$\downarrow$

Nothing

$f(1) = 1 - 3 + 3 - 2$

$f(1) = -1$

$f(2) = 8 - 12 + 6 - 2 = 0$

$f(0) = 0 - 0 + 0 - 2 = -2$

37

$y = x\sqrt{4-x^2}$ $\quad y' = x \cdot \cancel{\frac{1}{2}}(4-x^2)^{-\frac{1}{2}} \cdot -\cancel{2}x + \sqrt{4-x^2} \cdot 1$

$y = x(4-x^2)^{\frac{1}{2}}$ $\quad y = \frac{-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}$

$$y = \frac{-x^2+4-x^2}{\sqrt{4-x^2}} \Rightarrow y = \frac{-2x^2+4}{\sqrt{4-x^2}}$$

$\sqrt{4-x^2}$

$$0 = \frac{-2x^2+4}{\cancel{\sqrt{4-x^2}}} \cdot \cancel{\sqrt{4-x^2}}$$

$$0 = -2x^2+4$$

$-4 \qquad\qquad -4$

$$-4 = -2x^2$$

$$\boxed{x = \pm\sqrt{2} \quad , 2 + -2}$$

$f(x) = x\sqrt{4-x^2}$ $\quad$ $\underline{\text{Domain}}$ $[-2, 2]$

$f(\sqrt{2}) = \sqrt{2} \cdot \sqrt{4-\sqrt{2}^2}$

$= \sqrt{2} \cdot \sqrt{2}$

$f(\sqrt{2}) = 2$ (max)

$f(-\sqrt{2}) = -2$ (min)

$f(2) = 0$ (rel. min)

$f(-2) = 0$ (rel. max)

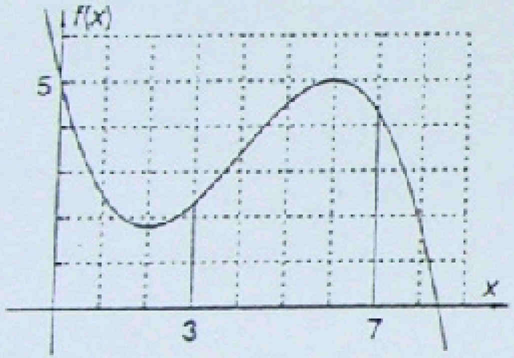
Name Max do this Group ________ Date ________

Exploration 29: The Mean Value Theorem

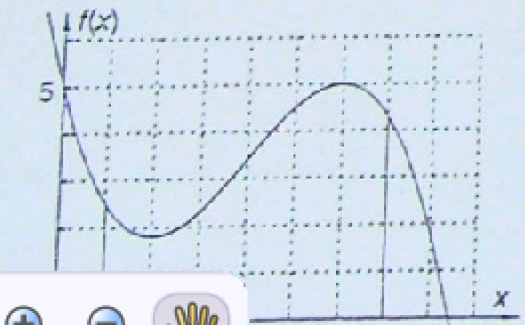
See Next page for #3, 4

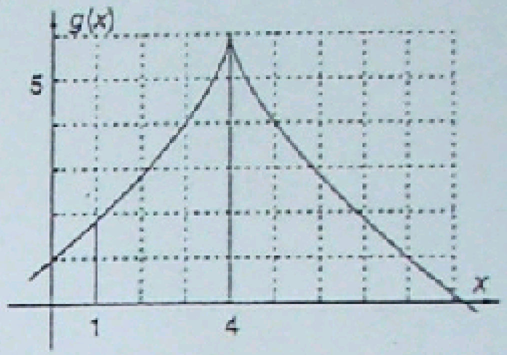
Objective: Without looking at the text, discover the hypotheses and conclusion of the mean value theorem.

1. For $f(x) = -0.1x^3 + 1.2x^2 - 3.6x + 5$, graphed below, there is a value of $x = c$ between 3 and 7 at which the tangent to the graph is parallel to the secant line connecting $(3, f(3))$ and $(7, f(7))$. Draw the secant line and the tangent line. Approximately what is the value of c?

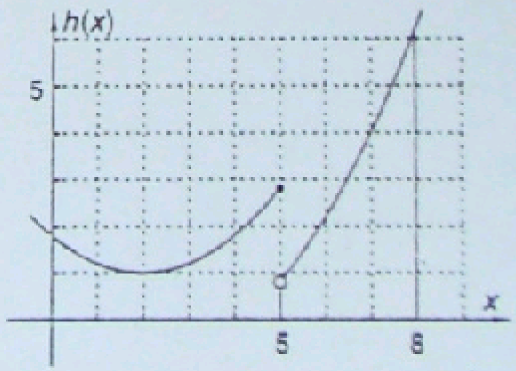


2. Function f in Problem 1 has *two* values of $x = c$ between $x = 1$ and $x = 7$ at which the tangent lines are parallel to the corresponding secant line. Draw these tangents on the graph below. Approximately what are the values of c?

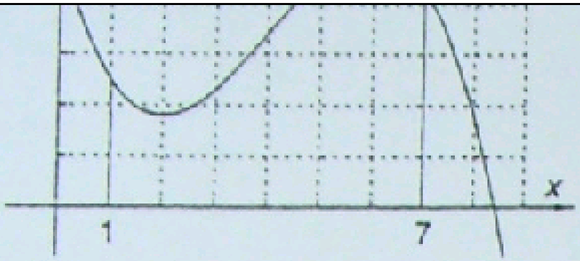




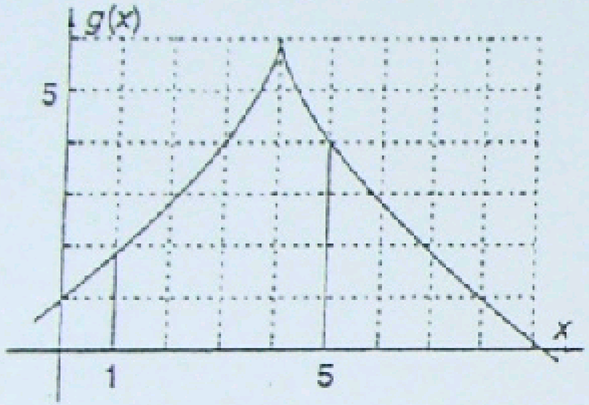
5. Function $h(x) = 0.2(x-2)^2 - \dfrac{|x-5|}{x-5}$ if $x \neq 5$, and $h(5) = 2.8$. Is h differentiable for all x in the *open* interval $(5, 8)$? Is h continuous at $x = 5$? Is there a value of $x = c$ in $(5, 8)$ for which $h'(c)$ equals the slope of the secant line connecting $(5, h(5))$ and $(8, h(8))$? Illustrate your answer.



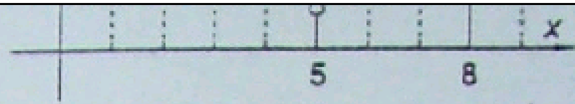
6. Function h in Problem 5 is differentiable on $(1, 5)$ and discontinuous at $x = 5$. Is there a point $x = c$ in



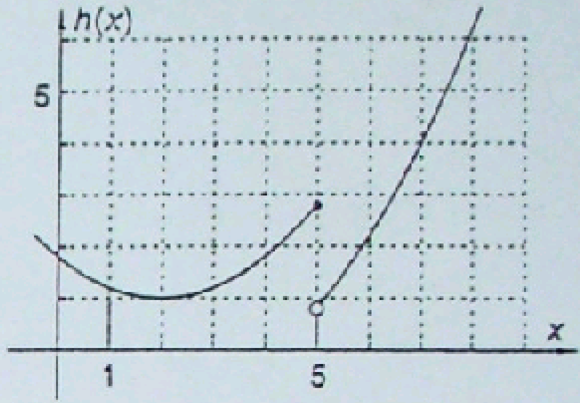
3. Function $g(x) = 6 - 2(x-4)^{2/3}$, graphed below, is not differentiable at $x = 4$. Is there a value of $x = c$ between $x = 1$ and $x = 5$ at which the slope of the tangent line equals the slope of the corresponding secant line? If so, draw it. If not, tell why not.



4. Function g in Problem 3 *does* have a value of $x = c$ between $x = 1$ and $x = 4$ where the tangent line parallels the corresponding secant line. This is true because the point at which the function is not differentiable occurs at the *endpoint* of the interval. Illustrate this fact on the graph in the next column. Approximately what is the value of c?



6. Function h in Problem 5 is differentiable on $(1, 5)$ and discontinuous at $x = 5$. Is there a point $x = c$ in the interval $(1, 5)$ at which $h'(c)$ equals the slope of the corresponding secant line? Illustrate your answer on the graph below.



7. The number $x = c$ in the above problems is the "mean" value referred to the mean value theorem. State the mean value theorem. Explain why the hypotheses are sufficient conditions but *not* necessary conditions.

Mean Value Theorem

"If $y=f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which $f'(c) = \dfrac{f(b)-f(a)}{b-a}$."

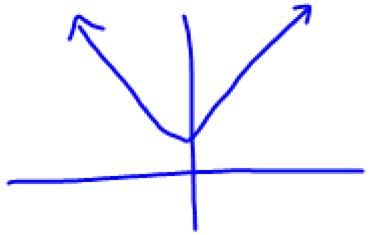
① Show that $f(x) = x^2$ satisfies the mean value thrm. on the interval $[0, 2]$ and find pt. c.

$\dfrac{f(b) - f(a)}{b - a} \rightarrow \dfrac{4 - 0}{2} = 2$ secant $\qquad f'(x) = 2x \qquad 2x = 2$

$x = 1$ so $c = 1$

② Explain why each of the following fails the mean value Thrm. on the interval $[-1, 1]$

ⓐ $f(x) = \sqrt{x^2} + 1 \longrightarrow |x| + 1$



ⓑ $f(x) = \begin{cases} x^3 + 3, & x < 1 \\ x^2 + 1, & x \geq 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = 4$

$\lim_{x \to 1^+} f(x) = 2$

Corollary 1:

"If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b)

1. If $f'(x) > 0$ at every pt (a, b) then $f(x)$ increases over $[a, b]$
2. If $f'(x) < 0$, decreases

where does $f(x) = x^3 - 4x$ increase, decrease?

$f'(x) = 3x^2 - 4$

$3x^2 - 4 > 0$

$3x^2 > 4$

$\sqrt{x^2} > \sqrt{\frac{4}{3}}$

$x < -\sqrt{\frac{4}{3}}$ $\quad$ $x > \sqrt{\frac{4}{3}}$

$3x^2 - 4 < 0$

$\sqrt{x^2} < \sqrt{\frac{4}{3}}$

$x > -\sqrt{\frac{4}{3}}$

$x < \sqrt{\frac{4}{3}}$

$-\sqrt{\frac{4}{3}} < x < \sqrt{\frac{4}{3}}$

4.2 #2, 4, 7, 10, 11, 14, 15, 18, 22, 25, 28